What is Claimed is:

1. A method executed in a computer system for producing a combined system of partial differential equations comprising:

representing each of a plurality of systems as an application mode modeling physical

5   quantities of said each system;

determining a representation of a partial differential equation system for each application

mode corresponding to one of said plurality of systems using at least one non-local coupling,

said at least one non-local coupling determining a value in at least one point depending on a

value from at least one other point; and

10   forming said combined system of partial differential equations using partial differential

equation systems associated with said plurality of systems.

2. The method of Claim 1 wherein said non-local variable defines a value from a first

15   domain in a first geometry to another domain in a second geometry.

3. The method of Claim 2, further comprising:

forming, for each of said first and second geometries, a system of partial differential

equations each having associated coupling variables.

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4. The method of Claim 2, wherein at least one of said partial differential equation

systems uses at least one local coupling.

5. The method of Claim 2, wherein said first and second geometries are the same.

6. The method of Claim 2, wherein said first and second geometries are different.

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7. The method of Claim 1, further comprising:

defining a non-local coupling wherein a value of a quantity on a boundary of a first

domain is referenced on parallel lines extending into said first domain.

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8. The method of Claim 2, further comprising:

defining a non-local coupling in which a boundary condition associated with said first

domain is defined using a value of an integral over a portion of one of: said first domain and said

second domain.

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9. The method of Claim 1, further comprising:

defining a non-local coupling using at least one of: a mapped variable and an integrated

variable.

10. The method of Claim 4, further comprising:

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defining a local coupling using at least one of: a basic variable, an auxiliary variable, and

a glued variable.

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11. The method of Claim 1, further comprising:

defining a non-local coupling variable using at least one of: an extrusion variable, a

5   projection variable, and a scalar coupling variable.

12. The method of Claim 1, further comprising:

determining a stiffness matrix by determining at least one of a Jacobian of a variable and

a value of a variable in accordance with a type of said variable wherein said combined system of

10   partial differential equations is in weak form, said stiffness matrix being a Jacobian matrix of a

residual vector with respect to a number of degrees of freedom, said Jacobian of a variable being

represented as at least one contribution determined in accordance with a number of degrees of

freedom; and

determining said residual vector by determining at least one of a Jacobian of a variable

15   and a value of a variable in accordance with a type of said variable wherein said combined

system of partial differential equations is in weak form.

13. The method of Claim 12, further comprising:

converting said combined system of partial differential equations from general form to

20   weak form.

14. The method of Claim 12, wherein said determining said stiffness matrix further comprises:

determining at least one of a Jacobian of a variable and a value of a variable in accordance with points included in a quadrature formula and with other points in accordance

5   with coupling variables.

15. The method of Claim 14, wherein said determining said residual vector further comprises:

determining at least one of a Jacobian of a variable and a value of a variable in

10   accordance with points included in a quadrature formula and with other points in accordance with coupling variables.

16. The method of Claim 1, further comprising:

determining a value of a variable in accordance with a type of said variable used in at

15   least one of said partial differential equation systems.

17. The method of Claim 16, wherein variables are recursively evaluated in accordance with variable type.

20   18. The method of Claim 17, wherein said determining a value of a variable in accordance with a type is used in determining at least one of a: stiffness matrix, constraint matrix, residual vector and a constraint residual vector.

19. The method of Claim 18, wherein said type is one of: a basic variable, an auxiliary variable, a glued variable, a mapped variable, and an integrated variable.

20. The method of Claim 1, further comprising:

5   determining a Jacobian of a variable in accordance with a type of said variable used in at least one of said partial differential equations wherein said Jacobian of a variable is represented in accordance with a number of degrees of freedom.

21. The method of Claim 20, wherein a Jacobian of a variable is recursively evaluated in 10 accordance with variable type.

22. The method of Claim 21, wherein said determining a Jacobian of a variable in accordance with a type is used in determining at least one of: a stiffness matrix, a residual vector, constraint residual vector, and a constraint matrix.

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23. The method of Claim 22, wherein said type is one of: a basic variable, an auxiliary variable, a glued variable, a mapped variable, and an integrated variable.

24. The method of Claim 1, wherein said at least one non-local coupling includes a 20 variable having a dependency on another variable at at least one distant point.

25. The method of Claim 24, wherein said other variable is in the same geometry as said variable.

26. The method of Claim 24, wherein said other variable is in a different geometry from said variable.

5   27. The method of Claim 4, wherein said local coupling includes a variable having a dependency only on values of other variables at the same point.

28. The method of Claim 1, further comprising:

defining a non-local coupling used in at least one of a: subdomain, boundary, edge, and point that obtains a value at one of: a subdomain, boundary, edge, and point.

29. The method of Claim 4, further comprising:

defining a local coupling using at least one of: an expression variable and a boundary coupled variable.

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30. The method of Claim 1, further comprising:

defining a non-local coupling wherein a value of an integral of a variable along parallel lines extending into a first domain is used on a boundary on said first domain.

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31. The method of Claim 1, further comprising:

defining a boundary condition on one boundary in terms of a value of a variable on

another boundary wherein said value is mapped in accordance with a coordinate transformation.

32. The method of Claim 1, further comprising:

5   defining a boundary condition in terms of a variable defined at a single point.

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33. A method executed in a computer system for assembling a finite element discretization of a system of weak partial differential equations comprising:

determining a stiffness matrix by evaluating at least one of a Jacobian of a variable and a value of a variable in accordance with a type of said variable included in said system, said

5   Jacobian of said variable being represented as at least one contribution in accordance with a number of degrees of freedom; and

determining a residual vector by evaluating at least one of a Jacobian of a variable and a value of a variable in accordance with a type of said variable included in said system, said Jacobian of said variable being represented as at least one contribution determined in accordance

10 with a number of degrees of freedom.

34.   The method of Claim 33, wherein said system includes at least one non-local coupling and said method further includes evaluating at least one variable in said non-local coupling in at least one of: said determining said stiffness matrix and said determining said

15 residual vector.

35. The method of Claim 33, wherein said system includes at least one local coupling and said method further includes evaluating at least one variable in said local coupling in at least one of: said determining said stiffness matrix and said determining said residual vector.

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36. The method of Claim 33, further comprising:

determining a constraint matrix by evaluating at least one of said Jacobian of a variable

and a value of a variable in accordance with a type of said variable included in said system

wherein said Jacobian of said variable is represented as at least one contribution in accordance

5   with a number of degrees of freedom.

37. The method of Claim 36, further comprising:

determining a constraint residual vector by determining a value of at least one variable

included in said system in accordance with a type of said at least one variable.

38. The method of Claim 37, wherein said type is one of: a basic variable, an auxiliary

variable, a glued variable, a mapped variable, an integrated variable.

39. The method of Claim 33, wherein said type is one of: a basic variable, an auxiliary

15   variable, a glued variable, a mapped variable, an integrated variable.

40. A computer program product for producing a combined system of partial differential equations comprising machine executable code for:

representing each of a plurality of systems as an application mode modeling physical quantities of said each system;

5   determining a representation of a partial differential equation system for each application mode corresponding to one of said plurality of systems using at least one non-local coupling, said at least one non-local coupling determining a value in at least one point depending on a value from at least one other point; and

forming said combined system of partial differential equations using partial differential

10   equation systems associated with said plurality of systems.

41. The computer program product of Claim 40, wherein said non-local variable defines a value from a first domain in a first geometry to another domain in a second geometry.

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42. The computer program product of Claim 41, further comprising machine executable code for:

forming, for each of said first and second geometries, a system of partial differential equations each having associated coupling variables.

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43. The computer program product of Claim 41, wherein at least one of said partial differential equation systems uses at least one local coupling.

44. The computer program product of Claim 41, wherein said first and second geometries are the same.

5   45. The computer program product of Claim 41, wherein said first and second geometries are different.

46. The computer program product of Claim 40, further comprising machine executable code for:

10   defining a non-local coupling wherein a value of a quantity on a boundary of a first domain is referenced on parallel lines extending into said first domain.

47. The computer program product of Claim 41, further comprising machine executable code for:

15   defining a non-local coupling in which a boundary condition associated with said first domain is defined using a value of an integral over a portion of one of: said first domain and said second domain.

48. The computer program product of Claim 40, further comprising machine executable

20   code for:

defining a non-local coupling using at least one of: a mapped variable and an integrated variable.

49. The computer program product of Claim 43, further comprising machine executable code for:

defining a local coupling using at least one of: a basic variable, an auxiliary variable, and a glued variable.

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50. The computer program product of Claim 40, further comprising machine executable code for:

defining a non-local coupling variable using at least one of: an extrusion variable, a projection variable, and a scalar coupling variable.

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51. The computer program product of Claim 40, further comprising machine executable code for:

determining a stiffness matrix by determining at least one of a Jacobian of a variable and a value of a variable in accordance with a type of said variable wherein said combined system of

15 partial differential equations is in weak form, said stiffness matrix being a Jacobian matrix of a residual vector with respect to a number of degrees of freedom, said Jacobian of a variable being represented as at least one contribution determined in accordance with a number of degrees of freedom; and

determining said residual vector by determining at least one of a Jacobian of a variable

20 and a value of a variable in accordance with a type of said variable wherein said combined system of partial differential equations is in weak form.

52. The computer program product of Claim 51, further comprising machine executable code for:

converting said combined system of partial differential equations from general form to weak form.

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53. The computer program product of Claim 51, wherein said machine executable code for determining said stiffness matrix further comprises:

machine executable code for determining at least one of a Jacobian of a variable and a value of a variable in accordance with points included in a quadrature formula and with other points in accordance with coupling variables.

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54. The computer program product of Claim 53, wherein said machine executable code for determining said residual vector further comprises:

machine executable code for determining at least one of a Jacobian of a variable and a value of a variable in accordance with points included in a quadrature formula and with other points in accordance with coupling variables.

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55. The computer program product of Claim 40, further comprising machine executable code for:

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determining a value of a variable in accordance with a type of said variable used in at least one of said partial differential equation systems.

56. The computer program product of Claim 55, wherein variables are recursively evaluated in accordance with variable type.

57. The computer program product of Claim 56, wherein said machine executable code for determining a value of a variable in accordance with a type is used in machine executable code for determining at least one of a: stiffness matrix, constraint matrix, residual vector and a constraint residual vector.

58. The computer program product of Claim 57, wherein said type is one of: a basic variable, an auxiliary variable, a glued variable, a mapped variable, and an integrated variable.

59. The computer program product of Claim 40, further comprising machine executable code for:

determining a Jacobian of a variable in accordance with a type of said variable used in at least one of said partial differential equations wherein said Jacobian of a variable is represented in accordance with a number of degrees of freedom.

60. The computer program product of Claim 59, wherein a Jacobian of a variable is recursively evaluated in accordance with variable type.

61. The computer program product of Claim 60, wherein said machine executable code for determining a Jacobian of a variable in accordance with a type is used in machine executable code for determining at least one of: a stiffness matrix, a residual vector, constraint residual vector, and a constraint matrix.

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62. The computer program product of Claim 61, wherein said type is one of: a basic variable, an auxiliary variable, a glued variable, a mapped variable, and an integrated variable.

63. The computer program product of Claim 40, wherein said at least one non-local coupling includes a variable having a dependency on another variable at at least one distant point.

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64. The computer program product of Claim 63, wherein said other variable is in the same geometry as said variable.

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65. The computer program product of Claim 63, wherein said other variable is in a different geometry from said variable.

66. The computer program product of Claim 43, wherein said local coupling includes a variable having a dependency only on values of other variables at the same point.

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67. The computer program product of Claim 40, further comprising machine executable code for:

defining a non-local coupling used in at least one of a: subdomain, boundary, edge, and point that obtains a value at one of: a subdomain, boundary, edge, and point.

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68. The computer program product of Claim 43, further comprising machine executable code for:

defining a local coupling using at least one of: an expression variable and a boundary coupled variable.

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69. The computer program product of Claim 40, further comprising machine executable code for:

defining a non-local coupling wherein a value of an integral of a variable along parallel lines extending into a first domain is used on a boundary on said first domain.

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70. The computer program product of Claim 40, further comprising machine executable code for:

defining a boundary condition on one boundary in terms of a value of a variable on

20 another boundary wherein said value is mapped in accordance with a coordinate transformation.

71. The computer program product of Claim 40, further comprising machine executable code for:

defining a boundary condition in terms of a variable defined at a single point.

72. A computer program product for assembling a finite element discretization of a system of weak partial differential equations comprising machine executable code for:

determining a stiffness matrix by evaluating at least one of a Jacobian of a variable and a value of a variable in accordance with a type of said variable included in said system, said

5   Jacobian of said variable being represented as at least one contribution in accordance with a number of degrees of freedom; and

determining a residual vector by evaluating at least one of a Jacobian of a variable and a value of a variable in accordance with a type of said variable included in said system, said Jacobian of said variable being represented as at least one contribution determined in accordance

10   with a number of degrees of freedom.

73.   The computer program product of Claim 72, wherein said system includes at least one non-local coupling and said computer program product further includes machine executable code for evaluating at least one variable in said non-local coupling in at least one of: said

15   machine executable code for determining said stiffness matrix and said machine executable code for determining said residual vector.

74. The computer program product of Claim 72, wherein said system includes at least one local coupling and said computer program product further includes machine executable code

20   for evaluating at least one variable in said local coupling in at least one of: said machine executable code for determining said stiffness matrix and said machine executable code for determining said residual vector.

75. The computer program product of Claim 72, further comprising machine executable code for:

determining a constraint matrix by evaluating at least one of said Jacobian of a variable and a value of a variable in accordance with a type of said variable included in said system

5   wherein said Jacobian of said variable is represented as at least one contribution in accordance with a number of degrees of freedom.

76. The computer program product of Claim 75, further comprising machine executable code for:

determining a constraint residual vector by determining a value of at least one variable

10 included in said system in accordance with a type of said at least one variable.

77. The computer program product of Claim 76, wherein said type is one of: a basic variable, an auxiliary variable, a glued variable, a mapped variable, an integrated variable.

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78. The computer program product of Claim 72, wherein said type is one of: a basic variable, an auxiliary variable, a glued variable, a mapped variable, an integrated variable.